ELEKTROSTATIK & ELEKTRODYNAMIK

Husk at $\boldsymbol{r}$ er observationspunktet, $\boldsymbol{r}'$ er ladningsposition og $\boldsymbol{\mathscr{r}} = \boldsymbol{r} - \boldsymbol{r}'$

Kilde	$\boldsymbol{E}(\boldsymbol{r})$	$V(\boldsymbol{r})$
Generelt	$\frac{1}{4\pi\varepsilon_0}\int_\nu \frac{\rho(\boldsymbol{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,\mathrm{d}\tau'$	$\frac{1}{4\pi\varepsilon_0}\int_\nu \frac{\rho(\boldsymbol{r}')}{\mathscr{r}}\mathrm{d}\tau'$
Punktladning	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}}$	$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
Kugleskal (radius R)	$\begin{cases}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}} & \text{for } r \geq R \\ 0 & \text{for } r < R\end{cases}$	$\begin{cases}\frac{1}{4\pi\varepsilon_0}\frac{Q}{r} & \text{for } r \geq R \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} & \text{for } r < R\end{cases}$
Uendelig Plan (i yz-plan) med overfladeladning σ	$\frac{\sigma}{2\varepsilon_0}\,\mathrm{sgn}(x)\hat{\boldsymbol{x}}$	$-\frac{\sigma\,\lvert x\rvert}{2\varepsilon_0}$
Dipol	$\frac{1}{4\pi\varepsilon_0}\frac{3(\boldsymbol{p}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}-\boldsymbol{p}}{r^3}$	$\frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{r}}}{r^2}$
Dipol hvor $\boldsymbol{p} \parallel \hat{\boldsymbol{z}}$	$\frac{1}{4\pi\varepsilon_0}\frac{p(3\cos\theta\hat{\boldsymbol{r}}+\sin\theta\hat{\boldsymbol{\theta}})}{r^3}$	$\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$

Tabel I: *Hvis kilden ikke er i origo skal man passe til på med vektorer og hvad* $\mathscr{r}$ *egentlig er.*

Elektrisk potentiale (gælder kun elektrostatik):

$$\boldsymbol{E} = -\nabla V \tag{1}$$

$$\therefore \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{\ell} = V(\boldsymbol{a}) - V(\boldsymbol{b}) \tag{2}$$

Gauss Lov – Integralform – "Gaussflade"

$$\oiint \boldsymbol{E}(\boldsymbol{r})\cdot \mathrm{d}\boldsymbol{a} = \frac{Q_{enc}}{\varepsilon_0} \tag{3}$$

Lysets Hastighed:

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{4}$$

Maxwells Ligninger (i differentialform):

$\nabla\cdot\boldsymbol{E} = \frac{\rho}{\varepsilon_0}$	Gauss Lov
$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}$	Faradays Lov
$\nabla\cdot\boldsymbol{B} = 0$	Divergens af $\boldsymbol{B}$
$\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{J} + \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t}$	Ampéres lov

Capacitor:

Generelt:

$$C \equiv \frac{Q}{\Delta V} \tag{5}$$

Af en pladekondensator:

$$C = \frac{A\varepsilon_0}{d} \tag{6}$$

Dipolmoment:

$$\boldsymbol{p} = \int_\nu \boldsymbol{r}'\rho(\boldsymbol{r}')\mathrm{d}\tau' \tag{7}$$

Hvis man har med punktladninger at gøre, hvor $\rho(\boldsymbol{r}')$ er en deltafunktion, reduceres udtrykket til:

$$\boldsymbol{p} = \sum_i q_i\boldsymbol{r}_i' \tag{8}$$

Kraftmoment, N, påført af feltet $\boldsymbol{E}$ på $\boldsymbol{p}$:

$$\boldsymbol{N} = \boldsymbol{p}\times\boldsymbol{E} \tag{9}$$

Kraften påført af feltet $\boldsymbol{E}$ på $\boldsymbol{p}$:

$$\boldsymbol{F} = (\boldsymbol{p}\cdot\nabla)\boldsymbol{E} \tag{10}$$

Elektrisk kraft:

$$\boldsymbol{F} = q\boldsymbol{E} \tag{11}$$

hvor q ladningen af ladningen bliver påvirket af kraften.

Arbejde og energi af et elektrisk felt:

$$W_{felt} = \int_a^b \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{\ell} = -\Delta U \tag{12}$$

$$W = \frac{1}{2}\int \rho V \mathrm{d}\tau = \frac{1}{2}\int \sigma V \mathrm{d}a = \frac{1}{2}\int \lambda V \mathrm{d}\ell \tag{13}$$

For punktladninger bliver det altså

$$W = \frac{1}{2}\sum_k q_k V(\boldsymbol{r}_k) \tag{14}$$

En kondensator:

$$W = \frac{1}{2}CV^2 \tag{15}$$

ELEKTRISK FELT I MATERIALER:

Polarisation:

$$\begin{aligned} &\boldsymbol{P} \equiv \textit{dipole moment per unit volume} \\ &\boldsymbol{p} = \boldsymbol{P}\mathrm{d}\tau' \end{aligned} \tag{16}$$

Overfladeladningstæthed af et dielektrikum:

$$\sigma_b \equiv \boldsymbol{P} \cdot \hat{\boldsymbol{n}} \tag{17}$$

Volumenladningstæthed af et dielektrikum:

$$\rho_b \equiv -\nabla \cdot \boldsymbol{P} \tag{18}$$

For lineær media gælder

$$\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E} \tag{19}$$

"Electric Displacement"

$$\begin{aligned} &\boldsymbol{D} \equiv \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} \\ &\nabla \cdot \boldsymbol{D} = \rho_f \end{aligned} \tag{20}$$

Gauss' lov for "electric displacement":

$$\oiint \boldsymbol{D}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{a} = Q_{f_{enc}} \tag{21}$$

. . . .

MAGNETOSTATIK

Kilde	$\boldsymbol{B}(\boldsymbol{r})$	$\boldsymbol{A}(\boldsymbol{r})$
Generelt	$\frac{\mu_0}{4\pi}\int \frac{\boldsymbol{I} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\mathrm{d}\ell'$	$\frac{\mu_0}{4\pi}\int \frac{\boldsymbol{J}(\boldsymbol{r}')}{\mathscr{r}}\mathrm{d}\tau'$
Leder ($\ell = \infty$) med jævn strøm $\boldsymbol{I} \parallel \hat{\boldsymbol{z}}$	$\frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\varphi}}$	$-\frac{\mu_0 I}{2\pi}\ln(s)\hat{\boldsymbol{z}}$
Loop med radius R ($B_z(0,0,z)$)	$\frac{\mu_0 I}{2}\frac{R^2}{(z^2+R^2)^{\frac{3}{2}}}$	
Spole med n vindinger per længde og radius R. Længde ℓ (Problem 5.11)	$\frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)$	

Tabel II: *Pas på med retningerne. Jeg bruger* $\boldsymbol{I} = I\boldsymbol{t}$ *hvor* $\boldsymbol{t}$ *tangerer ledningen.*

Magnetisk Vektorpotentiale:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{22}$$

Ampéres lov – integralform – "Amperian loop":

$$\begin{aligned} &\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I_{enc} \\ &\oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} = \Phi \end{aligned} \tag{23}$$

Flux:

$$\Phi = \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a} \tag{24}$$

Elektromotoriskkraft (emf):

$$\varepsilon = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{\ell} \tag{25}$$

Lorentz kraften:

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{26}$$

Kraften på en leder:

$$\boldsymbol{F} = \int \boldsymbol{I} \times \boldsymbol{B} \mathrm{d}\ell = \int I(\mathrm{d}\boldsymbol{\ell} \times \boldsymbol{B}) \tag{27}$$

For hhv. overladestrøm og volumenstrøm gælder

$$\boldsymbol{F} = \int (\boldsymbol{K} \times \boldsymbol{B}) \mathrm{d}a \tag{28}$$

og

$$\boldsymbol{F} = \int (\boldsymbol{J} \times \boldsymbol{B}) \mathrm{d}\tau \tag{29}$$

MAGNETISK FELT I LINEÆR MEDIA:

Magnetisation:

$$\boldsymbol{M} \equiv \textit{magnetic dipole moment per unit volume} \tag{30}$$
$$\boldsymbol{m} = \boldsymbol{M} \mathrm{d}\tau' \tag{31}$$

Bundede strøm på overfladen:

$$\boldsymbol{K} = \boldsymbol{M} \times \hat{\boldsymbol{n}} \tag{32}$$

Bundede strom i volumenet:

$$\boldsymbol{J} = \nabla \times \boldsymbol{M} \tag{33}$$

$\boldsymbol{H}$-feltet:

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M} \tag{34}$$

Ampéres lov for H-feltet:

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{\ell} = I_{f_{enc}} \tag{35}$$

og

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f \tag{36}$$

....

MATEMATIK

Gradientteoremet:

$$\int_{\boldsymbol{a}}^{\boldsymbol{b}} \nabla f \cdot \mathrm{d}\boldsymbol{\ell} = f(\boldsymbol{b}) - f(\boldsymbol{a}) \tag{37}$$

Divergensteoremet:

$$\int (\nabla \cdot \boldsymbol{A}) \mathrm{d}\tau = \oiint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{a} \tag{38}$$

Rotationsteoremet:

$$\int (\nabla \times \boldsymbol{A}) \cdot \mathrm{d}\boldsymbol{a} = \oint \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{\ell} \tag{39}$$

Gradient af $\mathscr{r}^{-1}$:

$$\nabla' \left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} = -\nabla \left(\frac{1}{\mathscr{r}}\right) \tag{40}$$

Tensornotation:

Levi-Civita:

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{for} \quad (i,j,k) = (1,2,3) \vee (3,1,2) \vee (2,3,1) \\ -1 & \text{for} \quad (i,j,k) = (2,1,3) \vee (1,3,2) \vee (3,2,1) \\ 0 & \text{for} \quad (i=j) \vee (i=k) \vee (j=k) \end{cases}$$

Vektoridentiteter vha. tensornotation

$$\begin{aligned} (\boldsymbol{v} \times \boldsymbol{w})_i &= \varepsilon_{ijk} v_j w_k \\ \boldsymbol{v} \cdot \boldsymbol{w} &= v_i w_i \\ \nabla \cdot \boldsymbol{v} &= \partial_i v_i \\ (\nabla \times \boldsymbol{v})_i &= \varepsilon_{ijk} \partial_j v_k \\ (\nabla \boldsymbol{v})_{ij} &= \partial_i v_j \\ ((\boldsymbol{v} \cdot \nabla) \boldsymbol{w})_i &= v_j \partial_j w_i \end{aligned} \tag{41}$$

Hyppige integraler:

$$\begin{aligned} &\int \frac{1}{\sqrt{a^2+x^2}} \mathrm{d}x = \ln\left(x + \sqrt{a^2+x^2}\right) = \sinh^{-1}\left(\frac{x}{a}\right) \\ &\int \frac{x}{\sqrt{a^2+x^2}} \mathrm{d}x = \sqrt{a^2+x^2} \\ &\int \frac{1}{(a^2+x^2)^{\frac{3}{2}}} \mathrm{d}x = \frac{x}{a^2\sqrt{a^2+x^2}} \\ &\int \frac{x}{(a^2+x^2)^{\frac{3}{2}}} \mathrm{d}x = \frac{-1}{\sqrt{a^2+x^2}} \end{aligned} \tag{42}$$

Præfiks	navn	$\log_{10}$
M	mega-	6
k	kilo-	3
-	-	0
m	milli-	-3
μ	mikro-	-6
n	nano-	-9
p	pico-	-12

Vektoranalyse

Infinitesimaler

Kartetisk:

$$\mathrm{d}\boldsymbol{\ell} = \mathrm{d}x\hat{\boldsymbol{x}} + \mathrm{d}y\hat{\boldsymbol{y}} + \mathrm{d}z\hat{\boldsymbol{z}} \tag{43}$$

Sfærisk:

$$\mathrm{d}\boldsymbol{\ell} = \mathrm{d}r\hat{\boldsymbol{r}} + r\mathrm{d}\theta\hat{\boldsymbol{\theta}} + r\sin\theta\mathrm{d}\varphi\hat{\boldsymbol{\varphi}} \tag{44}$$

Cylindrisk:

$$\mathrm{d}\boldsymbol{\ell} = \mathrm{d}r\hat{\boldsymbol{r}} + r\mathrm{d}\theta\hat{\boldsymbol{\theta}} + \mathrm{d}z\hat{\boldsymbol{z}} \tag{45}$$

Gradienten

Kartetisk:

$$\nabla\beta = \frac{\partial\beta}{\partial x}\hat{i} + \frac{\partial\beta}{\partial y}\hat{j} + \frac{\partial\beta}{\partial z}\hat{k} \tag{46}$$

Polær:

$$\nabla\beta = \frac{\partial\beta}{\partial r}\hat{i}_r + \frac{1}{r}\frac{\partial\beta}{\partial\theta}\hat{i}_\theta \tag{47}$$

Sfærisk

$$\nabla\beta = \frac{\partial\beta}{\partial r}\hat{i}_r + \frac{1}{r}\frac{\partial\beta}{\partial\theta}\hat{i}_\theta + \frac{1}{r\sin\theta}\frac{\partial\beta}{\partial\varphi}\hat{i}_\varphi \tag{48}$$

Gradienten af en vektor

$$\vec{v}\cdot\nabla\vec{v} = v_x\frac{\partial\vec{v}}{\partial x} + v_y\frac{\partial\vec{v}}{\partial y} + v_z\frac{\partial\vec{v}}{\partial z} \tag{49}$$

Divergens

Kartetisk:

$$\nabla\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \tag{50}$$

Polær:

$$\nabla\cdot\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} \tag{51}$$

Sfærisk

$$\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial\varphi}$$

Rotation

Kartetisk:

$$\nabla\times\vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{k} \tag{52}$$

Sfærisk:

$$\nabla\times\vec{v} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(v_\varphi\sin\theta) - \frac{\partial v_\theta}{\partial\varphi}\right)\hat{i}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial}{\partial r}(rv_\varphi)\right)\hat{i}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial\theta}\right)\hat{i}_\varphi \tag{53}$$

Cylindrisk:

$$\nabla\times\vec{v} = \frac{1}{r}\begin{vmatrix} \hat{i}_r & r\hat{i}_\varphi & \hat{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ v_r & rv_\varphi & v_z \end{vmatrix} \tag{54}$$

(Stregerne vil sige man skal tage determinanten af matricen)

Laplace

Kartetisk:

$$\nabla^2\beta = \frac{\partial^2\beta}{\partial x^2} + \frac{\partial^2\beta}{\partial y^2} + \frac{\partial^2\beta}{\partial z^2} \tag{55}$$

Sfærisk:

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\varphi^2} \tag{56}$$

Polær (planar):

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial\theta^2} \tag{57}$$