Termodynamik Formelsamling

Termodynamikkens Hovedsætninger

0. $(A \approx B) \wedge (B \approx C) \implies (A \approx C)$

1. $\mathrm{d}U = Q + W \overset{QS}{=} Q - P\mathrm{d}V$

2. $\Delta S_{\text{tot}} \geq 0,\ S = k_B \ln(\Omega)$

3. $(T \to 0) \implies (S \to 0)$

Termodynamiske Identiteter

$$\begin{aligned}
\mathrm{d}U &= T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N \\
\mathrm{d}H &= T\mathrm{d}S + V\mathrm{d}P + \mu\mathrm{d}N \\
\mathrm{d}F &= -S\mathrm{d}T - P\mathrm{d}V + \mu\mathrm{d}N \\
\mathrm{d}G &= -S\mathrm{d}T + V\mathrm{d}P + \mu\mathrm{d}N
\end{aligned}$$

Nogen Multipliciteter

Ideal Gas:

$$\Omega(U, V, N) = f(N) V^N U^{\frac{3N}{2}} \tag{1}$$

Einstein Solid:

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N-1)!} \tag{2}$$

Paramagnet:

$$\Omega(N_\uparrow, N_\downarrow) = \frac{N!}{N_\uparrow! N_\downarrow!} \tag{3}$$

Maxwell's Relationer

$$\begin{aligned}
\left(\frac{\partial T}{\partial V}\right)_{S,N} &= -\left(\frac{\partial P}{\partial S}\right)_{V,N} \\
\left(\frac{\partial T}{\partial P}\right)_{S,N} &= \left(\frac{\partial V}{\partial S}\right)_{P,N} \\
\left(\frac{\partial S}{\partial V}\right)_{T,N} &= \left(\frac{\partial P}{\partial T}\right)_{V,N} \\
\left(\frac{\partial S}{\partial P}\right)_{T,N} &= -\left(\frac{\partial V}{\partial T}\right)_{P,N}
\end{aligned}$$

Entropi og Energi

$$\begin{aligned}
\mathrm{d}S &\geq \frac{Q}{T} \\
\mathrm{d}S &= \frac{Q}{T}\ \text{(Kvasistatisk)} \\
U &= \frac{f}{2} N k_B T \\
\mathrm{d}U &= \frac{f}{2} N k_B \mathrm{d}T
\end{aligned}$$

Varme Kapaciteter

$$\begin{aligned}
C_V &= \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \\
C_P &= \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_{P,N}
\end{aligned}$$

Fri energi

$$\begin{aligned}
H &= U + PV \\
F &= U - TS \\
G &= U + PV - TS = \mu N
\end{aligned}$$

Nogen Fancy Ord

$$\begin{aligned}
(\mathrm{d}V = 0) &\equiv \text{Isokorisk} \\
(\mathrm{d}P = 0) &\equiv \text{Isobar} \\
(\mathrm{d}T = 0) &\equiv \text{Isotermisk} \\
(\mathrm{d}S = 0) &\equiv \text{Isentropisk} \\
(Q = 0) &\equiv \text{Adiabatisk}
\end{aligned}$$

Heat Engines

$$e = \frac{\text{benefit}}{\text{cost}} = \frac{W_{\text{out}}}{Q_H} = 1 - \frac{Q_C}{Q_H}$$
$$e \leq 1 - \frac{T_C}{T_H} = e_{\text{Carnot}}$$

Effektivitet af Ottomotoren

$$e = 1 - \left(\frac{V_{\max}}{V_{\min}}\right)^{\gamma - 1} \tag{4}$$

Arbejde langs isoterm (fra V_1 til V_2)

$$W = -Nk_BT\ln\left(\frac{V_2}{V_1}\right) \tag{5}$$

Arbejde langs adiabat

$$W = \frac{f}{2}Nk_B\Delta T = \frac{f}{2}(P_2V_2 - P_1V_1) \tag{6}$$

Varme langs en adiabat

$$Q = 0 \tag{7}$$

Varme langs en isoterm

$$Q = -W = \int_{V_1}^{V_2} P(V)\mathrm{d}V \tag{8}$$

Adiabatisk Proces:

$$PV^\gamma = konstant$$
$$VT^{\frac{f}{2}} = konstant$$

Faseovergang

$G_i = G_j \to \mathrm{d}G_i = \mathrm{d}G_j$ ved faselinjen (hvor i og j referer til de to faser)

$$\frac{\mathrm{d}P}{\mathrm{d}T} = \frac{\Delta S}{\Delta V} = \frac{L}{T\Delta V}$$

Statistisk Mekanik

$$Z \equiv \sum_s e^{-\beta E(s)} = \sum_E \Omega(E)e^{-\beta E}$$
$$\mathcal{P}(s) = \frac{e^{-\beta E(s)}}{Z}; \quad \sum_s \mathcal{P}(s) = 1$$
$$\langle X\rangle = \frac{1}{Z}\sum_s X(s)e^{-\beta E(s)}$$
$$\langle E\rangle = -\frac{\partial \ln(Z)}{\partial\beta} = -\frac{1}{Z}\frac{\partial Z}{\partial\beta}$$
$$U = \langle E\rangle$$
$$F = -k_BT\ln(Z)$$

Matematik

$$\ln(ab) = \ln(a) + \ln(b)$$
$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

Stirlingsapproksimation (gælder når $N \gg 1$)

$$N! \approx N^Ne^{-N}\sqrt{2\pi N}$$

og

$$\ln(N!) \approx N\ln(N) - N + \frac{1}{2}\ln(2\pi N)$$

Konstanter

$$k_B = 1.381 \times 10^{-23}\frac{\text{J}}{\text{K}} = 8.617 \times 10^{-5}\frac{\text{eV}}{\text{K}}$$
$$N_A = 6.022 \times 10^{23}$$
$$h = 6.626 \times 10^{-34}\text{Js} = 4.136 \times 10^{-15}\text{eVs}$$
$$1\text{atm} = 1.013\text{bar} = 1.013 \times 10^5\text{Pa}$$
$$1\text{eV} = 1.602 \times 10^{-19}\text{J}$$
$$1\text{u} = 1.661 \times 10^{-27}\text{kg}$$